Optimized Modelling of a Laboratory-scaled Water Distribution System

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ABSTRACT

In today's modern control system, optimization is essential in enhancing system capabilities and to achieve the "best" design relative to a set of prioritized constrains. This study aims to investigate the application of particle swarm optimization (PSO) in optimizing a PID controller that is developed for the mathematical models of laboratory-scaled water distribution system produced by the Prediction Error Minimization (PEM) in MATLAB. In this work, the priority is focused on improving the PID controller performance, in terms of the level control of a laboratory-scaled process system. This is achieved by determining the optimal PID controller parameters utilizing PSO tuning technique. From the results obtained, the selected mathematical model of the system are the sixth order and an eighth order model which yield zero steady-state errors when applied with the PID-PSO algorithm. The proposed method was more robust and efficient in improving the performance of the water level control.

Keywords: Optimization, Water Distribution System, Leven Sensor, Particle Swarm Optimization, Prediction Error Minimization, PID Controller

1. Introduction

Water resources in Malaysia are abundant and readily available, either for domestic use, or for industrial application. In a water distribution network, water is transferred from a natural resource to a water treatment plant to be treated and distributed to customers through a water distribution system. To ensure the adequacy of clean water to customers, a reliable water distribution system is needed to help control the pump, hence controlling the reservoir tank level, thus producing sufficient clean water to customers. Currently, water level control in Malaysia is controlled manually in the water tanks, thus a tedious and an inaccurate process are being implemented. Therefore, in this research, an automatic water level controller is proposed, to yield a high accuracy water level control as well as to extinguish water wastage in the water distribution network.

Currently, to meet the needs of daily activities in various domains, various applications are developed. Problem-solving techniques becomes more formidable and demanding. The major concerns in engineering problems are essentially maximizing earning by minimizing losses. Due to the diversified field of knowledge, optimization problems become more complex and sophisticated as science and technology develop. In recent years, metaheuristic optimization algorithms have emerged on top due to their distinct advantages over traditional algorithms such as linear programming, non-linear programming, etc., which are found to be efficient but contain some limitations and drawbacks. Since metaheuristics can solve multiple-objective, multiple solutions, and non-linear formulations, they are crucial in finding the best solution to a constantly growing number of complex real-world problems.

Even in an era where advanced control algorithms based on optimization procedure have reached a high level of maturity, utilization of Proportional Integral Derivative (PID) controllers are still significant in the industrial applications albeit the new emerging techniques. PID simple yet robust structure is easy to understand by engineers, hence can be applied more efficiently under practical conditions, compared to other advanced complex controllers. In PID control, the three gain parameters are the proportional gain (k_p), integral gain (k_i) and derivative gain (k_d), needs to be determined and tuned appropriately.

In this work, an optimized level of a laboratory-scaled water distribution system is determined using Particle Swarm Intelligence (PSO) which will be used to obtain the optimal parameter tuning of the PID controllers for the level control. The mathematical model of the system is obtained through the modeling technique of prediction error minimization (PEM). The optimized mathematical model can then be used to initiate the set point of the reservoir tank of the system, which reflects the volume required.

2. Water Distribution System: A Laboratory set-up

The laboratory set-up for the water distribution system, which is an upgrade to the system in [1] and [2] is given in figure 1. From figure 1, the laboratory set-up can control the distribution of water from the clear water tank to the reservoir tank, automatically, through programmable logic controller (PLC). The mathematical model obtained is based on controlling the level in the reservoir tank, to ensure sufficient supply of clean water.

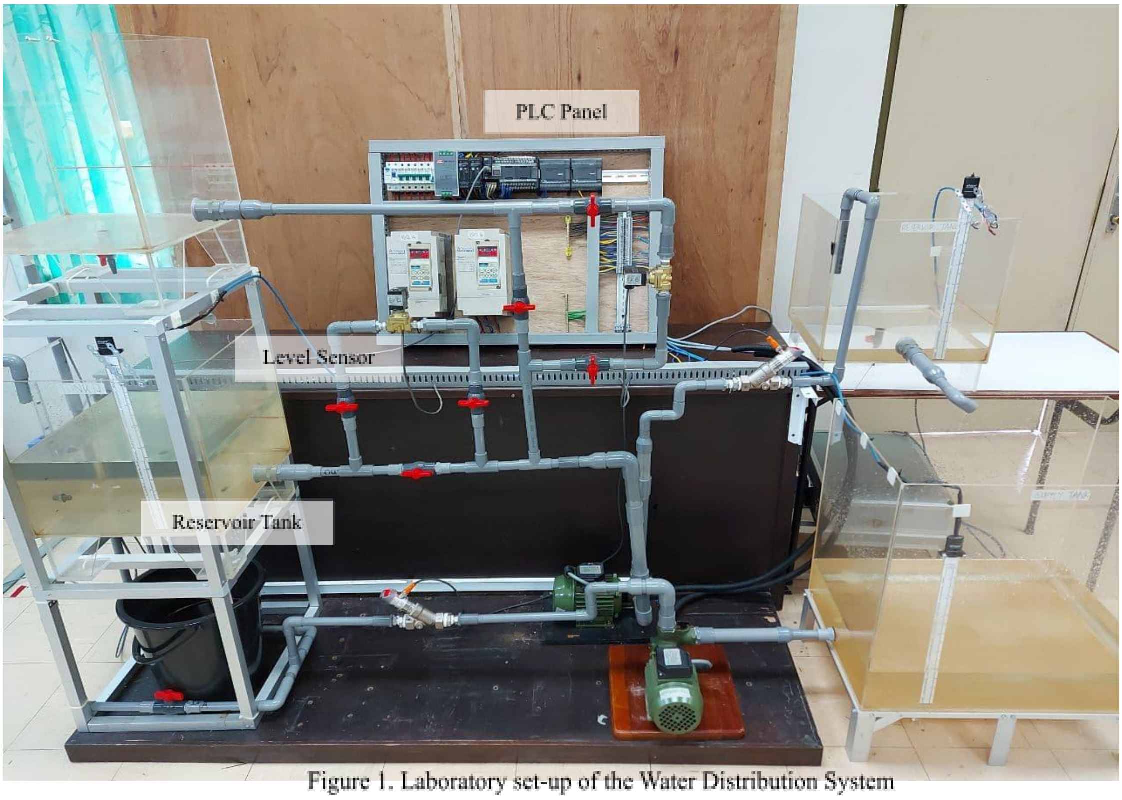

Figure 1. Laboratory set-up of the Water Distribution System

3. Prediction Error Minimization

Mathematical modelling is a method in representing a mathematical model in mathematical terms of the behavior of the real devices and objects. In the field of engineering and science, models are applied to predict future events. With an extensive library of solvers, the time of the upcoming simulation step can then be determined, and numerical methods can be applied to produce mathematic operations with different objects such as numbers, vectors, and matrices as well as to solve differential equations that represent the model.

In this study, the mathematical models of the laboratory-scaled process system, shown in figure 1, are computed by utilizing the measured input and output data of the water distribution system in the Excel spreadsheet. Prediction Error Minimization comes from an extensive family of parameter estimation methods that can be used on arbitrary model parameterizations [3]. PEM postulates a function of the prediction errors between a prediction model and a criterion. It possesses good asymptotic properties in which the estimation theory analyses the properties of the estimates under decent conditions. PEM utilizes numerical optimization to minimize the cost function, a weighted norm of the prediction error, defined as follows for scalar outputs:

$$V_N(G, H) = \sum_{t=1}^{N} e^2(t) \tag{1}$$

where $e(t)$ is the difference between the measured output and the predicted output of the original system. For a linear model, this error is defined by the following equation:

$$e(t) = H^{-1}(q)[y(t) - G(q)u(t)] \tag{2}$$

where $e(t)$ is a vector and the cost function $V_N(G, H)$ is a scalar value. The subscript N indicates that the cost function is a function of the number of data samples where for larger values of N, yields more accurate output. The previous equation is more complex for multiple-output models. From Table 1, the model order of the system is found to be stable.

Table 1. Best fits for mathematical models using PEM

Order	MSE Filtered	% Best Fits Filtered
2	3.70×10^{-4}	99.94869
3	1.75×10^{-6}	99.99647
4	5.20×10^{-8}	99.99939
5	7.57×10^{-9}	99.99977
6	3.88×10^{-9}	99.99983
7	9.14×10^{-9}	99.99975
8	2.82×10^{-8}	99.99955
9	1.38×10^{-7}	99.99943
10	1.38×10^{-7}	99.99901

From Table 1, the 8th order mathematical model is utilized to develop the optimized PID-control algorithm. The 6th order system in state-space is given as

$$G_6(s) = \frac{-2.597\times10^{-7}s^5+1.352\times10^{-6}s^4-3.83\times10^{-7}s^3+1.908\times10^{-6}s^2-3.16\times10^{-8}s+6.13\times10^{-7}}{s^6+0.05952s^5+0.05418s^4+0.002461s^3+0.0005276s^2+1.024\times10^{-5}s+7.039\times10^{-8}}$$

Meanwhile, the transfer function of the 8th order system in state-space is given below

$$G_8(s) = \frac{8.684\times10^{-7}s^7-3.139\times10^{-6}s^6+5.552\times10^{-6}s^5-7.453\times10^{-6}s^4+6.231\times10^{-6}s^3-2.953\times10^{-6}s^2-1.738\times10^{-7}s+1.02\times10^{-7}}{s^8+0.36s^7+0.1587s^6+0.02725s^5+0.005873s^4+0.0005082s^3+5.095\times10^{-5}s^2+1.457\times10^{-6}s+1.292\times10^{-8}}$$

4. PID Control

PID stands for Proportional, Integral and Derivative. Its versatile function as a controller has been used in industrial control applications for a long time. PID controllers are the addition of three terms with each of the terms yields individual error value e between the input and the output. The relation between them is defined by the following equation [4]:

$$Output = K_p \times e(t) + K_i \times \int_0^t e(t)dt + K_d \times \frac{de}{dt} \tag{3}$$

where K_p, K_i and K_d are the P, I and D parameters respectively.

A standard PID controller's transfer function can be denoted as either in "parallel form" given by equation (4) or the "ideal form" given by equation (5)

$$G(s) = K_p + K_i \frac{1}{s} + K_d \quad (4)$$

$$= K_p(1 + \frac{1}{T_i s} + T_d s) \quad (5)$$

where K_p is the proportional gain, K_i is the integral gain, K_d is the derivative gain, T_i is the integral time constant, and T_d is the derivative time constant.

The significant application of PID controllers are the capability in improving the dynamic response of a system as well as minimizing the steady-state error. Generally, the proportional gain reduces the steady-state error, however, fails in ensure the system's stability. On the other hand, the integral gain, forces the system to have zero-steady state errors, hence may cause instability. Lastly, the derivative controller, K_d is capable in improving the transient response, although might cause system instability, similar to the integral gain. Optimization technique such as Particle Swarm Optimization (PSO) can be employed to the controller to obtain the optimal gain parameters which can be applied to the mathematical model of the system.

5. PID-PSO Controller Algorithm

The components of a PID controller along with the plant block which contains the mathematical model of the system are illustrated in figure 3. Simulink can be employed to simulate a physical system such as the lab-scaled water distribution system. Generally, it can simulate the complete control system, including the control algorithm that associates with the physical plant. Simulink is essential for generating the approximate solutions of mathematical models that may be difficult to solve "*by hand*".

In general, in MATLAB, optimization is applied to Simulink models to adjust variables defining the parameters of the simulation. In this study, the PSO is utilized to optimize the controller of the system using similar approach. MATLAB workspace variables are utilized to define the parameters that are changing, including initial conditions or input signals to the model. With the initialization of PSO algorithm, the simulation can be run using the commands in simulink to generating the model outputs. PSO will be further discussed in the next section of this paper.

Figure 2. Feedback Control system with PID Controller and Cost Function

1.1 Performance Estimation of PID Controller

The PID controller tries to minimize the system error by calculating the error value $e(t)$ as the difference between a desired setpoint and a measured process variable. The performance criteria also known as the objective functions are mainly based on measures of the system error. Table 2 shows the objective functions of the PID controller design.

Table 2. Performance Estimation of a PID Controller [5]

Name of Criterion	Formula
Integral of Absolute Error (IAE)	$IAE = \int_0^\infty \|e(t)\| \, dt$
Integral of Square Error (ISE)	$ISE = \int_0^\infty e(t)^2 \, dt$
Integral of Time-weighted Absolute Error (ITAE)	$ITAE = \int_0^\infty t\|e(t)\| \, dt$

In this paper, to validate the proposed PID-controller, the objective function in the time domain was used as proposed by [7]. A performance index must always be positive or zero. The best system is defined by the system that can minimize the index significantly. The objective functions are multiplied together along with the stability of the mathematical model as a multi-objective function as shown in (6). The stability is defined by the location of the poles and zeros of the mathematical model inside the root locus plot.

$$F = IAE * ISE * ITAE * Stability \quad (6)$$

where F is the fitness function for PSO to decide which solution is better and replace the previous solution.

6. PSO-PID Tuning

6.1 Overview of PSO Algorithm

In 1995, James Kennedy and Russell Eberhart brilliantly developed the Particle Swarm Optimization (PSO) which has become of the most popular and significant nature-inspired metaheuristic optimization algorithms. Its development proposed other variates to shine in solving practical issues related to optimization albeit emergence of PSO as a promising algorithm solving optimization problems in the field of science and engineering [7]. By applying PSO, necessary parameters can be obtained to minimize a given objective or a fitness function.

Generally, PSO is said to simulate the behaviors of bird flocking and fish schooling in such a way that the said group is randomly hunting for food in an area. There is only a single piece of food in the area being searched. The PSO is initialized with a set of random particles (solutions) and the search for optima begins by updating the generations. In each iteration, two best values are selected to update each particle with the first one being the best solution (fitness) it has achieved. The fitness value is also stored. This value is known as "*pbest*". Another best value that is tracked by the PSO is a global best known as "*gbest*" [6].

6.2 Implementation of PSO-PID Tuning

In this study, the developed PID controller utilizes PSO algorithm in improving transient response of the water distribution system, i.e. PSO-based PID controller (PSO-PID). The PSO algorithm was utilized to determine three optimal controller parameters K_p, K_i and K_d such that the controlled system could acquire a good step response output. The 8th order model is used in this paper to show the significance of PSO method. To initiate the PSO method for searching the best controller parameters, the "individual" is used to replace the "particle" and the "population" is used to describe the "group". Supposed there are three parameters K_p, K_i and K_d which composed of an individual K by $K = [K_p, K_i, K_d]$, hence K consists of three members in an individual [8]. The real values will be assigned to these members. By the algorithm procedure, if there are n individuals in a population, then the dimension would be $n \times 3$. A set of good control parameters can achieve a good step response and result in minimization of performance index in time domain including percentage overshoot (OS), settling time (t_s), rise time (t_r) and steady-state error (E_{ss}).

7. Results

The first step of PSO is the initialization. Proper initialization can produce maximum and decent results. The initialization information used is presented in Table 3.

Table 3. PSO Initialization Parameters

Description	Values
Population size (n)	50
Maximum iteration (i)	100
Dimension of problem (v)	4
Maximum inertia (w_max)	0.9
Minimum inertia (w_min)	0.4
Cognitive learning and social learning ($c_1 = c_2$)	0.9

Utilizing iterations the criteria in Table 3, the convergence plot for the sixth and eight order are presented in figure 3(a) and figure 3(b) respectively.

Figure 3. Convergence plot of PSO-PI controller for i = 100 of the (a) 6th order system and (b) 8th order system

Meanwhile the parameter gains for PID controller for the sixth order and the eighth order are given in figure 4(a) and figure 4(b) respectively.

Figure 4. PID Controller Parameters using PSO for (a) sixth order system and (b) eight order system

Figure 5. PSO-PID Step response for (a) sixth order and (b) eighth order

Meanwhile, the step response of the PSO tuned PID-controller was also evaluated based on the rise time (t_r), settling time (t_s), percentage overshoot (OS) and fitness value. The plot for the step response of the PSO-PID controller for the sixth and eighth order model are shown in figure 5(a) and figure 5(b) respectively.

From figure 5, the step response of the PSO-PID controller (red) is compared with the original system (blue) for both the sixth and eight order system. It can be observed that PSO was able to optimize the PID controller to obtain the optimal gain parameters to achieve the desired set-points, with zero steady-state errors.

8. Conclusion

From the results, the proposed method to optimize the PID controller shows significant difference compared to the conventional Ziegler-Nichols method when it comes to optimization. The PSO-based PID controller was able to produce a good step response with no steady-state error. Fundamentally, the objective to optimize a controller to obtain a good step response for the system was achieved. The results that have been obtained from the simulation illustrate how PSO is significant in optimizing a controller. The PSO algorithm can perform an efficient search to obtain the optimal gain parameters.

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