# Relative Measure of Digital Colour by utilizing the Linearity of CIELUV Space

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## ABSTRACT

In the field of colour science and image processing, quantifying colours into meaningful information can be hard because the perception of colour is an individual experience and it is difficult to measure. This research proposed a method of comparing colours perceptually based on the CIELUV colour space. The Luv Similarity was created after observing the movement of the point of the same colour in the CIELUV space at different external brightness. This angular linearity can be utilize to create a colour measurement that are immune to the lighting effect. Our method combines the perceptually uniform Euclidean distance with its angular measurement and normalize the two components according to Weber contrast expression. This algorithm can judge and map the similarity of colour between two images while minimizing the lighting effect.

## 1. Introduction

Creating a colour space that is equivalent to visual differences has been the goal of colour scientists. Using sRGB space to measure the matric distance of colour will result in linear and digitally true value results. This is because sRGB colour space is a representation of different light frequencies in the digital format. Nothing wrong with using a linear distance of sRGB colour spaces for colour matching, but the results can be harsh depending on the situation. sRGB colour space matching is based on the value represented by the colour digitally while human understanding of colour is fuzzier. The 2 mostly used uniform colour spaces are CIELAB and CIELUV [1] since they are nearly uniform when compared to the chromaticity diagram [2]. These 2 were proposed by the International Commission on Illumination (CIE) [3] and were intended as a perceptually uniform space. CIELAB and CIELUV have different strengths and weaknesses depending on the application. CIELAB works better on reflective surfaces (printing industry) while CIELUV is better for self-illuminated colours (monitors) [4]. It is suggested that CIELUV is more stable for saturation and widely used for information visualization.

CIELUV was being underappreciated, there is rarely a discussion on the colour space especially on measuring colour difference or similarity. There are not many formulations and corrections were made since its first introduction, unlike its twin CIELAB. CIELUV colour space was created for self-luminated or colour display, while CIELAB was made for reflective surfaces and dyes. It is time for CIELUV to make it come back since most of the images these days are digitalized and no longer printed. CIELUV consist of 3 component which is L, u and v. L is the perceptual lightness, while u and v is the other 2 axis that represents the colour or hue space that mimics the human vision. To turn RGB to CIELUV, the sRGB first need to be converted into the XYZ (Rec. 709) colour spaces, which is a standard for image encoding and signal characteristics developed by ITU-R (International Telecommunication Union Radiocommunication Sector) [5]. sRGB can be converted into XYZ with D65 white point using the matrix below:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} \quad (1)$$

Once the XYZ components were calculated, L, u and v components can be derived using the formulas given:

$$L^* = \begin{cases} 116Y^{1/3} - 16 & for\ Y > \left(\frac{29}{3}\right)^3 \\ \left(\frac{29}{3}\right)^3 Y & for\ Y \leq \left(\frac{29}{3}\right)^3 \end{cases} \quad (2)$$

$$u' = \frac{4X}{X+15Y+3Z} \quad (3)$$

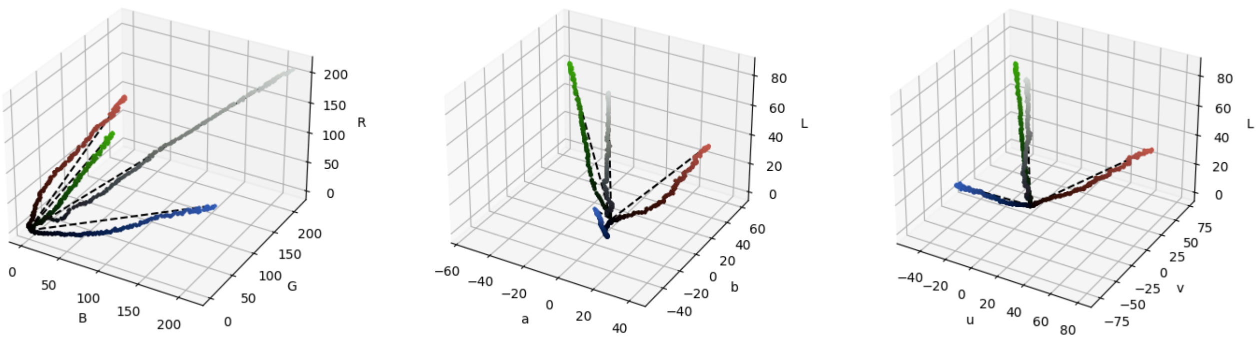
$$v' = \frac{9X}{X+15Y+3Z} \quad (4)$$

$$u^* = 13L(u' - 0.19793943) \quad (5)$$

$$v^* = 13L(v' - 0.46831096) \quad (6)$$

## 2. Properties of CIELUV

Colours or hues are human perceptions of light waves bouncing off a surface, and if no light is reflected, humans perceive it as black. Human brains are very good at adjusting their colour perception under different lighting intensities. This section shows how different colours at different brightness are represented in the colour spaces. This test was conducted by illuminating a white light on a piece of paper with matte red, green, blue and white colours. Every region of colour on the paper is at the same distance from the light source and receives the same amount of brightness. The brightness gradually increased and multiple pictures of the subject were captured starting from no light until it reaches maximum brightness. The pixel value of every colour was plotted on the RGB, CIELAB and CIELUV colour space.




a) b) c)

Figure 1. The rate of change of intensity in colour space, a) RGB. b) CIELAB. c) CIELUV.

**Table 1.** Angular distance of each colour with regression line in degree

| Colour Vector | sRGB | CIELAB | CIELUV |
|---|---|---|---|
| Red | 12.9413 | 17.0596 | 5.4376 |
| Green | 7.7355 | 8.1904 | 3.8013 |
| Blue | 12.0131 | 17.8055 | 4.3646 |
| White | 3.3414 | 10.7039 | 8.8851 |
| Mean distance | 9.0078 | 13.4398 | 5.6221 |

From figure 1., all colours started from the origin during zero lighting for all 3 colour spaces. The colours moved outward to correspond to their colour region as the external brightness gradually increased. A regression line was drawn connecting the origin point to the final point of each colour indicating the ideal path of the same colour at different brightness. Table 1, shows the angular distance measured between each point with its regression line of each colour. The small mean angular distance proves that the arrangement of colour points in CIELUV has the most linear rate of change of intensity compared to other types of colour spaces. This is a useful property when measuring the similarity of colours at different intensities because the same colour should have a small angular distance regardless of its brightness.

## 3. Colour Similarity

The Euclidean measures between two points on these uniform colour spaces are supposed to represent the perceptual colour difference. But there are also multiple equations for measuring the distance of colour that also include hue, chroma and rotation correction on top of the Euclidean formula. The additional calculation on the distance formula makes the results harder to scale since the reference distance is no longer standard to the colour space used. The Euclidean or Delta-E of CIELUV can be defined as:

$$\Delta E_{uv} = \sqrt{(L^*{}_1 - L^*{}_2)^2 + (u^*{}_1 - u^*{}_2)^2 + (v^*{}_1 - v^*{}_2)^2} \quad (7)$$

The distance metric of CIELUV is attributed to the difference in hue and brightness. For CIELAB, there are multiple formulations (CIE94, CIEDE2000, CMC I:c) that includes lightness, chroma and hue correction after some perceptual non-uniformity was detected on the basic Euclidean distance. Unfortunately, CIELUV was not as popular as CIELAB, since there are not many extensions or corrections published on $\Delta E_{uv}$. This study aims to make use of the linearity of the colour point of CIELUV space in making a colour similarity formula where it minimizes the effect of external brightness when comparing the tested image with the reference image. Euclidean similarity and angular similarity are based on a simple inversion of Weber contrast expression, where the Weber contrast is equal to the size of perceived change relative to background stimuli. There are two similarity equations used which are Euclidean similarity and angular similarity:

$$S_{euc} = 1 - \frac{\Delta E_{uv}}{d_{ref}} \quad (8)$$

$$S_{ang} = 1 - \frac{\theta_{uv}}{a_{ref}} \quad (9)$$

The angular distance, $\theta_{uv}$, can be calculated using the smallest angle between the 2 hues. But hue difference alone may cause inconsistency measurement with any point near the greyline. To overcome this, angular distance where either chroma value that is less than Just Noticeable Difference, $JND$, will be zero out.

$$\Delta h = atan2(v^*{}_1, u^*{}_1) - atan2(v^*{}_2, u^*{}_2) \quad (10)$$

$$\Delta h = \begin{cases} \Delta h - 2\pi & if\ \Delta h > \pi \\ \Delta h + 2\pi & if\ \Delta h < \pi \end{cases} \quad (11)$$

$$k_C = tanh\left(\frac{\sqrt{u^*{}_1{}^2 + v^*{}_1{}^2}}{1.6}\right) \times tanh\left(\frac{\sqrt{u^*{}_2{}^2 + v^*{}_2{}^2}}{1.6}\right) \quad (12)$$

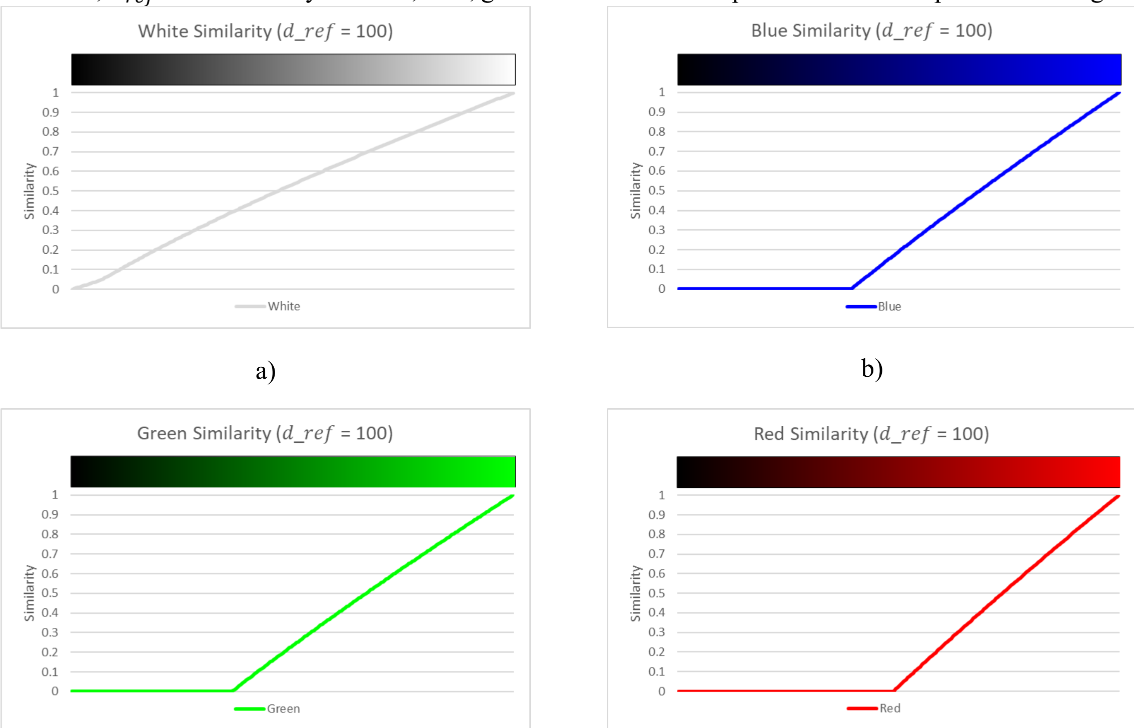
$$\theta_{uv} = k_C \times \Delta h \quad (13)$$

Chroma constant, $k_C$, is the multiplication between two hyperbolic functions, it acts as the activation function for the angular distance. In the tanh function, the chroma value is divided by 1.6 so that it will yield a result of 0.95 at chroma 2.9, (the common value of $JND$ for CIELUV [6]). The similarity value of $S_{euc}$ and $S_{ang}$ that is below than zero may be clip to zero since negative similarity can be consider as no similarity. Luv Similarity is the product of Euclidean and angular similarity:

$$S_{luv} = S_{euc} \times S_{ang} \quad (14)$$

## 4. Reference Distance

In this experiment, we test the Euclidean similarity by using the distance of black to white (100) in CIELUV as the reference distance, $d_{ref}$. The similarity of white, blue, green and red were compared with their respective colour gradient.




a) b)

c) c)

Figure 2. Euclidean similarity using constant reference distance ($d_{ref}$=100). a) White. b) Blue. c) Green. d) Red.

Figure 2., every colour has a different rate of similarity, the only colour that has a full range of similarity across the colour gradient is white. While similarity values for blue, green and red can only be measured on the higher part of the colour gradient. We can see that this is not true since the accent of the tested colour can be seen even in the lower part of the colour gradient. The reason for this is that the distance of blue, green, and red from the origin in the colour space is longer than the reference distance used. Using only one type of reference distance may cause an unfair measurement for other colours, and a small reference may cause a harsh and sensitive measurement on a wider colour gradient. The distance of gradient to black differs based on the colour vector, reference distance should be able to change its value depending on the position of the colour tested. To overcome this, we find the polynomial equation that best fits the vector of white, blue, green, yellow and red colours to its distance to the origin. The proposed polynomial reference distance expression can be written as:

$$d_{ref} = 183.319349 - 83.319381L^* + 30.504766u^* + 25.438910v^* - 120.527103L^*u^*v^* \quad (15)$$

The $L^*$, $u^*$and $v^*$ components in the reference distance formula are the normalised vector of the midpoint between 2 tested colours in CIELUV. Input vectors may also be either one of the tested colours, but the similarity formula will no longer be symmetry ($S_{euc}(I_1, I_2) \neq S_{euc}(I_2, I_1)$). So, we suggest using the average normalised vector as the result is more stable.

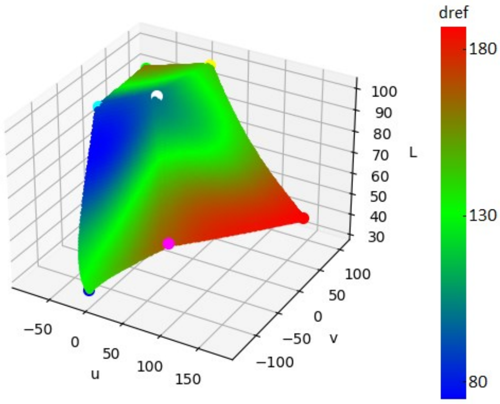
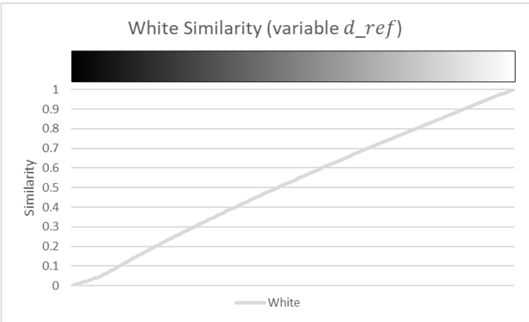
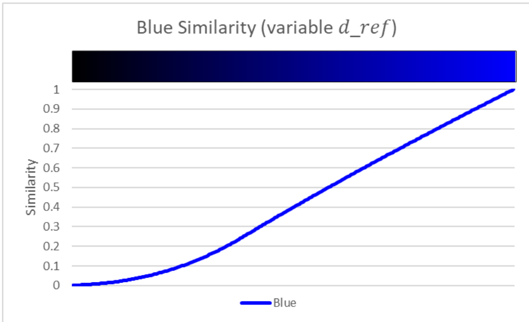



Figure 4.Estimated $d_{ref}$ in the CIELUV




a)

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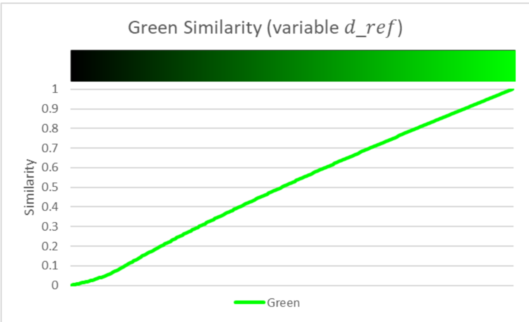


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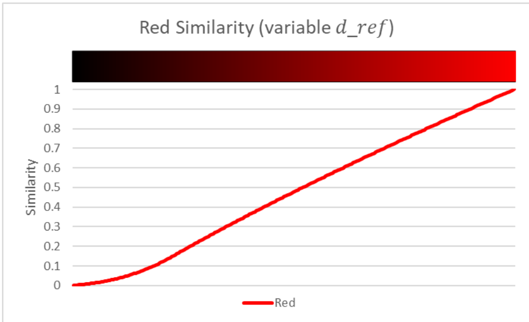
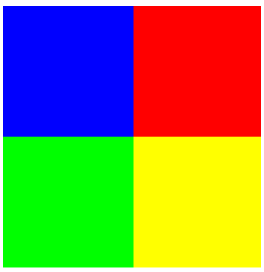


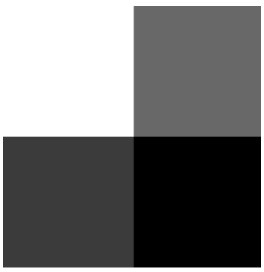

c)

Figure 4. Euclidean similarity using estimated reference distance. a) White. b) Blue. c) Green. d) Red.

A similar experiment of measuring Euclidean similarity between colours with their respective gradient was conducted, but in this experiment, the reference distance used was taken from the proposed polynomial equation. From figure 4., all colours have a proper range of similarity across the colour gradient to black.



a)



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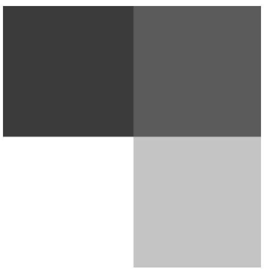
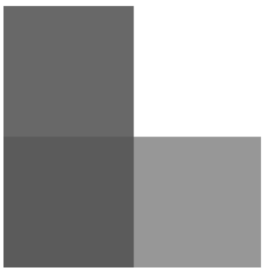


c)



d)

Figure 5. Angular similarity using constant reference angle ($a_{ref}$=3.142). a) Input image. b) Blue. c) Green. d) Red.

Figure 5. shows the result of the angular similarity between the input image with blue, green and red. The similarity result is between 0 (black) to 1 (white) being 1 as a high similarity and 0 with no similarity. All of the angular position of the colours in the CIELUV are perceptually accurate, based on how green are closer to yellow than blue to yellow. Even though the angular similarity using a constant Pi as the reference angle is correct, we would like to remove minor similarities between the different colours. The motivation for this is because colours such as blue, green, yellow and red are considered psychological primary, meaning that these are pure colours in the human mind and other colours are just psychological mixes [8]. Therefore, yellow and green should not have any similarity scores since these two should be perceptually different.

The angular position of colour in the CIELUV varies based on the hue position, thus one angular constant for reference may not be sufficient for measuring angular similarity. Thus, we fitted a polynomial that handles the difference in the range of hue based on the psychological primary colour vectors. The 3rd-degree polynomial of angular reference is as follows:

$$a_{ref} = 1.12522175 - 0.47827603h_{uv} + 0.04969178{h_{uv}}^2 + 0.04356436{h_{uv}}^3 \quad (16)$$

Estimated reference angles are calculated using the CIELUV hue value of the average between 2 inputs. Similar to the proposed reference distance, using only either one of the hue inputs will cause an asymmetry result. By applying the polynomial reference angle in the similarity formula, Figure 11., we can see that blue, green and red are not only properly highlighted but also able to minimize the similarity score of the other colour. Yellow and green do not have a high similarity, unlike previous angular similarity experiments, as they should be, corresponding with the psychological primary theory.

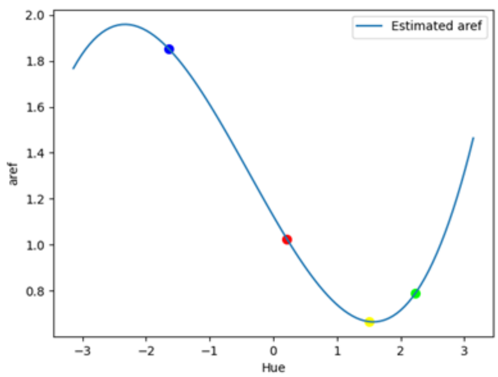



Figure 10. Estimated $a_{ref}$ on every angle.

a) b) c) d)

Figure 11. Angular similarity using estimated reference angle. a) Input image. b) Blue. c) Green. d) Red.

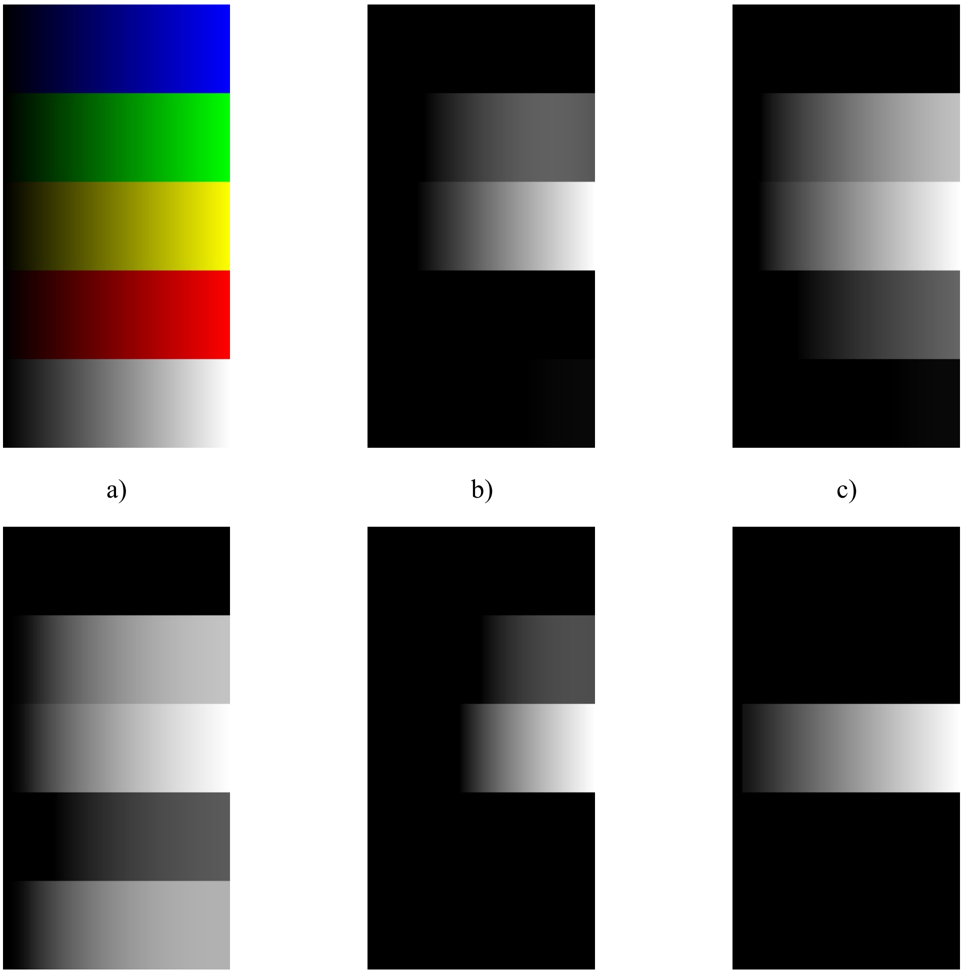
## 5. Luv Similarity

Luv Similarity, $S_{luv}$, is a proposed formulation for measuring the similarity between two colours that make use of the linearity of colour coordinate in the CIELUV at different brightness by combining Euclidean similarity and angular similarity. Euclidean similarity with estimated reference distance is responsible for computing the gradient similarity, measuring the lightness difference relative to the distance of the tested colour at full lightness to the origin. While angular similarity with estimated reference angle is in charge of finding the hue similarity inspired by the psychological primaries.

a) b)

Figure 12. Luv Similarity as colour detection. a) input image. b) similarity result.

In this experiment, we compared the result of our proposed Luv Similarity against the widely used colour difference formulation. Delta-E of CIE76, CIE94, CIEDE2000 and CMC l:c are in distance measurement, the results of Delta-E are scaled and threshold according to their local distance of black and white. The input image consists of the blue, green, yellow, red and white colour gradients evaluated in terms of similarity with the yellow colour.



a) b) c)

d) e) f)

Figure 13. Colour similarity with yellow. a) Input image. b) CIE76. c) CIE94. d) CIEDE2000. e) CMC l:c. f) Luv Similarity.

The result of the colour similarity test, Figure 13., shows that all previous Delta-E does not properly handle the hue and luminance difference. A small portion of the similarity of different colours is visible, especially on the green gradient. The reference distance for the Delta-E may be adjusted to reduce the similarity of other colours, but it will reduce the similarity at the same colour gradient. Our proposed formulation can remove the similarity between the 4 basic colours while preserving the similarity across the same colour gradient.

## 6. Conclusion

We have shown that the colour point in the CIELUV colour space has the most linear movement at different brightness compared to other types of perceptually uniform colour spaces. These properties can be used in assessing the similarity of colours since a similar colour should fall under the same vector. We proposed a similarity measure that minimizes the effect of lightness on the tested colours by incorporating the value of Delta-E with angular distance. The gradient distance differs based on the direction of the vector tested, we proposed an equation that controls the reference distance value. We also normalize the hue difference using our proposed reference angle formula, the idea behind this is to make the angular similarity obey the psychological primaries, where blue, green, yellow and red have zero similarity. We compared Luv Similarity with the commonly used Delta-E (CIE76, CIE94, CIEDE2000, CMC l:c) in the gradient/hue similarity test. The result shows that our proposed Luv Similarity can grade the similarity across the gradient while removing the similarity of different hues better than the other methods. We believe that this colour similarity measure will be able to be used in many types of applications that involve comparing or detecting colours.

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